Electromagnetic Waves.

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Origin of electromagnetic waves.

Oersted - 1820

* Electric charges in motion through a conductor is always accompanied with magnetic field around it.
* Michel Faraday 1831
  When magnetic field is changed around a closed ckt, an induced emf is generated.
* Electric current & magnetic field cannot be separated from each other, rather they co-exist & are co-related with each other.
* In 1861 James clerk Maxwell, purely on mathematical basis, predicted existance of electromagnetic waves & such waves can be radiated by accelarating charges.
* Hertz discovered E.M. waves in 1887.
* Faraday's law shows that changing magnetic field give rise to an electric field.
* Ampear-Maxwell law shows that changing electric field give rise to magnetic field.

Characteristics of Electromagnetic waves.

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1. Electromagnetic waves are produced whenever electric charges are accelerated. (antenna)
2. Electromagnetic waves propagate in the form of varying electric & magnetic field. These two fields are $\perp^r$ to each other & also $\perp^r$ to direction of propagation of E.M. waves.
3. E.M. waves can travel through vacuum.
4. E.M. waves can be reflected, refracted, transmitted or absorbed depending on nature of surface & frequency of wave.
5. E.M. waves travel in vacuum with velocity c given by —

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3\times10^8 \, m/s$$

& in the material medium, its velocity is

$$v = \frac{1}{\sqrt{\mu \epsilon}}$$

where μ is absolute permeability of medium & ϵ is absolute permittivity of medium.

(6) Energy of electromagnetic wave is divided equally between electric field & magnetic field.

(7) The cross-product $\vec{E} \times \vec{B}$ always gives direction of propagation of E.M waves.

(8) EM waves carry momentum. Hence they are able to exert pressure on surface where they strike. Tail of comet is always directed away from sun because pressure is exerted by sun rays on molecules evaporating from comet.

(9) E.M. wave transport energy from one region to other. The rate of energy flow per unit area or power flow per unit area is denoted by $\vec{S}$ known as Poynting vector

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \vec{E} \times \vec{H}$$

(10) Ratio of amplitudes of electric field & magnetic field is always constant & it is equal to velocity of E.M. waves.

$$C = \frac{E}{B} = 3\times10^{8} \text{ m/s}.$$

It means that when either electric or magnetic field varies with time, the other field is induced in space around it. The net effect is that an electromagnetic disturbance is produced due to changing electric & magnetic fields. This disturbance propagates in the form of electromagnetic wave.

$$\oint E \cdot dA = \frac{q}{\epsilon_0} \qquad \oint B \cdot dA = 0$$

$$\oint E \cdot dl = -\frac{d\phi_B}{dt} \qquad \oint B \cdot dl = \mu_0 I + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\nabla \times (\nabla \times \bar{E}) = \nabla \times \left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \bar{B})$$

Substituting for $\nabla \times B$ from eqn (5)

$$\nabla \times \nabla \times \bar{E} = -\frac{\partial}{\partial t}\left[\mu\sigma E + \mu\epsilon \frac{\partial E}{\partial t}\right]$$

$$\nabla \times \nabla \times \bar{E} = -\mu\sigma \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} \longrightarrow (6)$$

Applying identity for triple cross product i.e. to LHS

$$\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B}(\bar{A} \cdot \bar{C}) - \bar{C}(\bar{A} \cdot \bar{B})$$

$$\therefore \nabla \times \nabla \times \bar{E} = \nabla(\nabla \cdot \bar{E}) - \bar{E}(\nabla \cdot \nabla)$$

$$= 0 - \nabla^2 \bar{E} \longrightarrow (6)$$

$$\therefore -\nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\therefore \nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\epsilon \frac{\partial^2 E}{\partial t^2} \longrightarrow (7)$$

Electromagnetic wave equations in a conducting medium.

Let us consider a conducting medium of electrical conductivity σ & having finite permeability μ & permittivity ϵ, suppose medium is free from charged sources, hence charge density $\rho = 0$. In such case Maxwell's eq^{ns} can be written as -

$$\nabla \cdot \bar{D} = \epsilon \bar{\nabla} \cdot \bar{E} = 0 \quad \text{or} \quad \nabla \cdot E = 0 \longrightarrow (1)$$

$$\nabla \cdot B = 0 \longrightarrow (2)$$

$$\nabla \times \bar{E} = -\frac{\partial B}{\partial t} \longrightarrow (3)$$

$$\nabla \times \bar{B} = \mu\left[\bar{J} + \epsilon \frac{\partial \bar{E}}{\partial t}\right] = \mu\left[\sigma \bar{E} + \epsilon \frac{\partial \bar{E}}{\partial t}\right] \quad \because \bar{J} = \sigma \bar{E}$$

$$\nabla \times \bar{B} = \mu\sigma \bar{E} + \mu\epsilon \frac{\partial \bar{E}}{\partial t} \longrightarrow (4)$$

Wave eq^n for Electric field ($\bar{E}$) :-

Let us consider eq^n (3)

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

In order to eliminate $\bar{B}$ from above eq^n take curl on both sides of eq^n.

$$\therefore \nabla \times (\nabla \times \bar{E}) = \nabla \times \left(-\frac{\partial \bar{B}}{\partial t}\right)$$

Using ~~vector triple pro~~ identity for cross product of three vector

$$\bar{A}\times(\bar{B}\times\bar{C}) = \bar{B}(\bar{A}\cdot\bar{C}) - \bar{C}(\bar{A}\cdot\bar{B})$$

we can write

$$\cancel{\nabla\times\nabla\times\bar{E} = -\frac{\partial}{\partial t}(}$$

$$\nabla(\nabla\cdot\bar{E}) - \nabla^2\bar{E} = -\frac{\partial}{\partial t}(\nabla\times\bar{B})$$

Now $\nabla\cdot E = 0$ from eqⁿ ① & $(\nabla\times B) = \mu\sigma\bar{E} + \mu\epsilon\frac{\partial}{\partial}$ from eqⁿ ④

$$\therefore\ +\nabla^2\bar{E} = +\frac{\partial}{\partial t}\left[\mu\sigma\bar{E} + \mu\epsilon\frac{\partial\bar{E}}{\partial t}\right]$$

$$\nabla^2\bar{E} = \mu\sigma\frac{\partial\bar{E}}{\partial t} + \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2} \longrightarrow (5)$$

This is wave eqⁿ for electric field vector $(\bar{E})$ in conducting medium of conductivity σ.

Now, if medium is non conducting i.e. $\sigma = 0$, then above wave eqⁿ reduces to

$$\nabla^2 E = \mu\epsilon\frac{\partial^2 E}{\partial t^2} \longrightarrow$$

$$\nabla^2\bar{E} - \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2} = 0 \longrightarrow (6)$$

Electromagnetic wave equations in a conducting medium.

Let us consider conducting medium of electrical conductivity σ & having finite permeability μ & permittivity $\in$. Let us assume that medium is free from charge source, thus charge density $\rho = 0$. In this situation, Maxwell's eq^{ns} can be written as —

$$\nabla \cdot D = \in \nabla \cdot E = 0 \quad \therefore \bar{\nabla} \cdot \bar{E} = 0 \longrightarrow (1)$$

$$\nabla \cdot \bar{B} = 0 \longrightarrow (2)$$

$$\nabla \times \bar{E} = -\frac{\partial B}{\partial t} \longrightarrow (3)$$

$$\nabla \times \bar{B} = \mu \bar{J} + \mu \in \frac{\partial \bar{E}}{\partial t} \longrightarrow (4)$$

$$\nabla \times \bar{B} = \mu \left[\sigma \bar{E} + \in \frac{\partial \bar{E}}{\partial t} \right] \longrightarrow (5)$$

Wave eq^n for electric field E can be obtained from eq^n (3) by eliminating B. Taking curl of eq^n (3)

Electromagnetic wave eqⁿ in free space.

For free space $\sigma = 0$, $\rho = 0$, $\bar{J} = 0$, $\mu_r = 1$ & $\epsilon_r = 1$.

But $\mu = \mu_0 \cdot \mu_r = \mu_0 \cdot 1 = \mu_0$

& $\epsilon = \epsilon_0 \epsilon_r = \epsilon_0 \cdot 1 = \epsilon_0$.

In such case Maxwell's eqⁿ can be written as —

$\nabla \cdot \bar{D} = \epsilon_0 \nabla \cdot \bar{E} = 0 \quad \therefore \nabla \cdot \bar{E} = 0 \rightarrow (1)$

$\nabla \cdot \bar{B} = 0 \rightarrow (2)$

$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \rightarrow (3)$

$\nabla \times \bar{B} = \mu_0 \epsilon_0 \frac{\partial \bar{E}}{\partial t} \rightarrow (4)$.

Wave eqⁿ for electric field

$\nabla \times \nabla \times \bar{E} = \nabla \times \left(-\frac{\partial \bar{B}}{\partial t}\right)$

$\underbrace{\nabla(\nabla \cdot \bar{E})}_{0} - \nabla^2 \bar{E} = -\frac{\partial}{\partial t}(\nabla \times \bar{B})$

$+\nabla^2 \bar{E} = +\frac{\partial}{\partial t}\left(\mu_0 \epsilon_0 \frac{\partial \bar{E}}{\partial t}\right)$

$\boxed{\therefore \nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0} \rightarrow (5)$

Wave eqⁿ for mag field

$\nabla \times \nabla \times \bar{B} = \nabla \times \left[\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right]$

$\nabla(\nabla \cdot B) - \nabla^2 B = \mu_0 \epsilon_0 \frac{\partial}{\partial t}(\nabla \times E)$

$-\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial t}\right)$

$-\nabla^2 B = -\mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$

$\boxed{\therefore \nabla^2 B - \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} = 0} \rightarrow (6)$

Eqⁿ (5) & eqⁿ (6) are called as three dimensional wave equation for $\bar{E}$ & $\bar{B}$.

Any function satisfying eqⁿ (5) or eqⁿ (6) describes wave. The square root of the quantity that is the reciprocal of coefficient of the time derivative

field

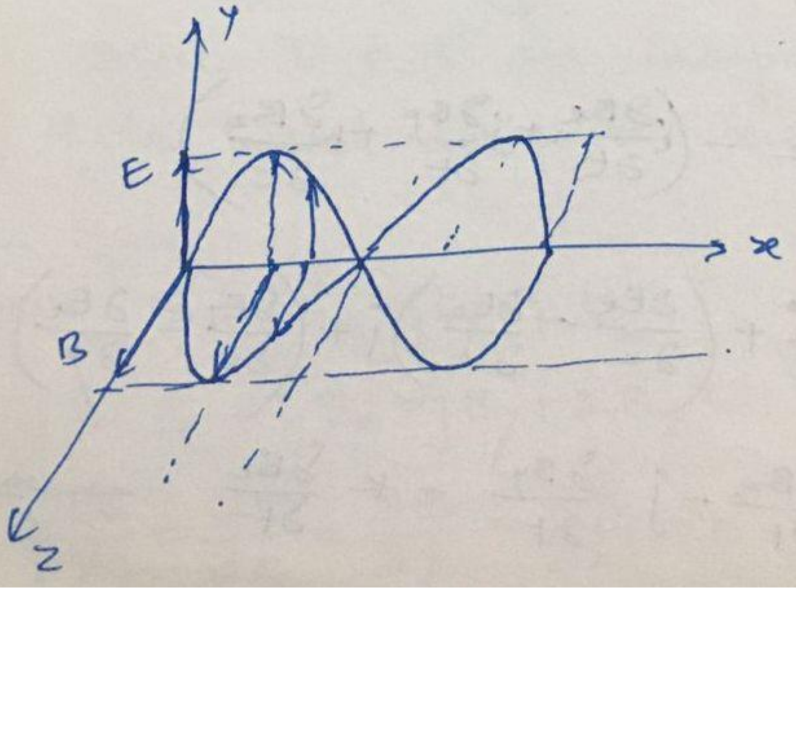
gives the velocity of wave. The above eq^ns represent variation of electric & magnetic field in space & time. It also gives velocity of these fields in space. 46

The velocity of propogation of these fields can be written as.

$$V = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{(4\pi \times 10^{-7}\ Wb/Am^2)(8.9 \times 10^{-12}\ c^2/Nm^2)}}$$

$$= 3.00 \times 10^{8}\ m/s = c \quad \text{velocity of light.}$$

Thus electromagnetic fields propogate in free space at speed of light. Emergence of speed of light from time independent constant quantities μ_0 & ϵ_0 is the most important achivements of Maxwell's eq^ns.



Wave eqⁿ for magnetic field ($\bar{B}$)

Let us consider eqⁿ (4)

$$\nabla \times \bar{B} = \mu \sigma \bar{E} + \mu \epsilon \frac{\partial E}{\partial t}$$

In order to eliminate $\bar{E}$ from above eqⁿ, take curl on both sides.

$$\nabla \times (\nabla \times B) = \nabla \times \left[\mu \sigma E + \mu \epsilon \frac{\partial E}{\partial t} \right]$$

$$\nabla (\nabla \cdot \bar{B}) - \nabla^2 \bar{B} = \mu \sigma (\nabla \times \bar{E}) + \mu \epsilon \frac{\partial}{\partial t} (\nabla \times \bar{E})$$

Now

$$\nabla \cdot B = 0 \quad \& \quad \nabla \times E = -\frac{\partial B}{\partial t}$$

$\therefore$

$$-\nabla^2 B = \mu \sigma \left(-\frac{\partial B}{\partial t} \right) + \mu \epsilon \frac{\partial}{\partial t} \left(-\frac{\partial B}{\partial t} \right)$$

$$+\nabla^2 B = +\left[\mu \sigma \frac{\partial B}{\partial t} + \mu \epsilon \frac{\partial^2 B}{\partial t^2} \right]$$

$$\therefore \nabla^2 B = \mu \sigma \frac{\partial B}{\partial t} + \mu \epsilon \frac{\partial^2 B}{\partial t^2} \longrightarrow (7)$$

This is wave eqⁿ for magnetic field $\bar{B}$ in conducting medium of conductivity σ.

Now if medium is non-conducting i.e. $\sigma = 0$, above eqⁿ reduces to

$$\nabla^2 \bar{B} = \mu \epsilon \frac{\partial^2 B}{\partial t^2} \qquad \boxed{\nabla^2 B - \mu \epsilon \frac{\partial^2 B}{\partial t^2} = 0}$$

(47)

$$\nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \mu_0 \epsilon_0 \left(\hat{i} \frac{\partial E_x}{\partial t} + j \frac{\partial E_y}{\partial t} + k \frac{\partial E_z}{\partial t} \right)$$

$$\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right) \hat{i} + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right) k + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) k$$

$$= \hat{i} \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t} + \hat{j} \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t} + k \mu_0 \epsilon_0 \frac{\partial E_z}{\partial t} \longrightarrow (4)$$

Let us assume that E.M. waves are plane waves propagating in x-direction. For such waves the vector field $\bar{E}$ & $\bar{B}$ is independent of y & z & displacement of these ~~waves~~ fields are functions of x & t only.

Since E & $\bar{B}$ are independent of y & z, their partial derivative w.r.t. y & z are zero.

Here $\bar{E} = \hat{i} E_x + j E_y + k E_z$

& $B = \hat{i} B_x + j B_y + k B_z$

& $\frac{\partial E}{\partial y} = \frac{\partial E}{\partial z} = \frac{\partial B}{\partial y} = \frac{\partial B}{\partial z} = 0$.

polarized

Maxwell's eq^n for plane electromagnetic waves in free space (i.e. vacuum)

For free space $\rho = 0$, $\sigma = 0$, $\mu = \mu_0$ & $\epsilon = \epsilon_0$

Maxwell's eq^{ns} can be written in vector form as —

(a) $\nabla \cdot \bar{E} = 0$

Now $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

& $\bar{E} = \hat{i}E_x + \hat{j}E_y + \hat{k}E_z$

$$\therefore \nabla \cdot E = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\hat{i}E_x + \hat{j}E_y + \hat{k}E_z\right)$$

$$\nabla \cdot \bar{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \longrightarrow (1)$$

Ill^y

$$\nabla \cdot \bar{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial E_z}{\partial z} \longrightarrow (2)$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -\left(i\frac{\partial B_x}{\partial t} + j\frac{\partial B_y}{\partial t} + k\frac{\partial B_z}{\partial t}\right)$$

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)i + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)j + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)k$$

$$= -i\frac{\partial B_x}{\partial t} - j\frac{\partial B_y}{\partial t} - k\frac{\partial B_z}{\partial t} \longrightarrow (3)$$

Since we are considering only the oscillatory motion of $\bar{E}$ & $\bar{B}$, a constant E_x & B_x will have no effect on the wave propagation. Hence they can be assumed to be zero. i.e. $E_x = 0$ & $B_x = 0$.

Substituting these values in eqn ③ & comparing x, y, z components on both sides we get

$$\underbrace{\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)}_{0} i + \left(\frac{\partial E_x}{\partial z} - \underset{0}{\frac{\partial E_z}{\partial x}}\right) j + \left(\frac{\partial E_y}{\partial x} - \underset{0}{\frac{\partial E_x}{\partial y}}\right) k$$

$$= -i\frac{\partial B_x}{\partial t} + j\frac{\partial B_y}{\partial t} - k\frac{\partial B_z}{\partial t}.$$

$$\cancel{i\,\partial B_x} \quad + \frac{\partial E_z}{\partial x}\cancel{j} = +\frac{\partial B_y}{\partial t}\cancel{j}$$

$$\therefore \frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t} \longrightarrow (9)$$

$$\text{IIIly} \quad \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \longrightarrow (10)$$

Similarly by substituting $E_x = 0$, $B_x = 0$ & partial derivatives w.r.t. y & z equal to zero we get

$$\left(\underbrace{\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}}_{0}\right)i + \left(\underbrace{\frac{\partial B_x}{\partial z}}_{0} - \frac{\partial B_z}{\partial x}\right)j + \left(\frac{\partial B_y}{\partial x} - \underbrace{\frac{\partial B_x}{\partial y}}_{0}\right)k$$

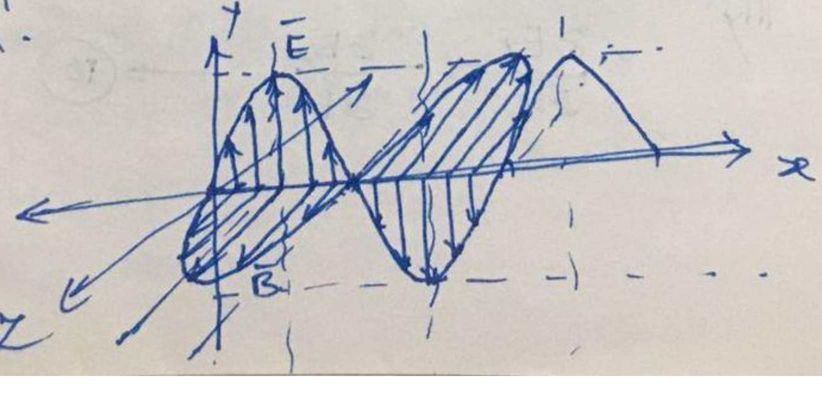
$$= \mu_0 \epsilon_0 \left[\underbrace{i\frac{\partial E_x}{\partial t}}_{0} + j\frac{\partial E_y}{\partial t} + k\frac{\partial E_z}{\partial t}\right]$$

$$-\frac{\partial B_z}{\partial x} j = \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t} j$$

$$\therefore \frac{\partial B_z}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E_y}{\partial t} \longrightarrow (11)$$

$$\frac{\partial B_y}{\partial x} = \mu_0 \epsilon_0 \frac{\partial E_z}{\partial t} \longrightarrow (12)$$

Eq^{ns} (9), (10) (11) & (12) are four wave eq^{ns} for plane polarized wave in free space i.e. vacuum.



$\therefore \frac{\partial E_y}{\partial y} = \frac{\partial E_z}{\partial z} =$

$$\frac{\partial E_y}{\partial y} = 0, \frac{\partial E_z}{\partial z} = 0, \frac{\partial B_y}{\partial y} = 0 \text{ \& } \frac{\partial B_z}{\partial z} = 0$$

Substituting this in eq^n (1) & (2).

$$\frac{\partial E_x}{\partial x} = 0 \rightarrow (5)$$

$$\frac{\partial B_x}{\partial x} = 0 \rightarrow (6)$$

Substituting partial derivatives of $\bar{E}$ & $\bar{B}$ w.r.t. y & z in eq^n (3) & (4) we get

$$-\frac{\partial B_x}{\partial t} = 0. \quad \therefore \frac{\partial B_x}{\partial t} = 0 \rightarrow (7)$$

$$\mu_0 \epsilon_0 \frac{\partial E_x}{\partial t} = 0 \quad \therefore \frac{\partial E_x}{\partial t} = 0 \rightarrow (8)$$

From eq^n (5), (6), (7) & (8) we conclude that x-component of either $\bar{E}$ or $\bar{B}$ do not vary with space-coordinate or time i.e. E_x & B_x are const.

Thus

$$\bar{E}\cdot\bar{B} = (iE_x + jE_y + kE_z)\cdot(iB_x + jB_y + kB_z)$$

$$EB\cos\theta = E_x B_x + E_y B_y + E_z B_z$$

But $E_x = 0$ & $B_x = 0$

$$\therefore EB\cos\theta = E_y B_y + E_z B_z$$

But from eq^n (13) we have $E_y B_y + E_z B_z = 0$

$$\therefore EB\cos\theta = 0$$

~~As E & B~~

$$\therefore \cos\theta = 0$$

$$\therefore \theta = \frac{\pi}{2} \text{ radians}$$

Thus electric field $\bar{E}$ & magnetic field $\bar{B}$ are at right angle to each other.

Wave eqⁿ for plane polarized E.M. wave can be obtained by taking $\frac{\partial}{\partial x}$ of eqⁿ 9 & $\frac{\partial}{\partial t}$ of eqⁿ 12

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$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial^2 B_y}{\partial t \partial x} \longrightarrow (13)$$

$$\frac{\partial^2 B_y}{\partial t \partial x} = \mu_0 \epsilon_0 \frac{\partial^2 E_z}{\partial t^2} \longrightarrow (14)$$

From above eqⁿˢ we can write.

$$\frac{\partial^2 E_z}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_z}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial x^2} - \mu_0 \epsilon_0 \frac{\partial^2 E_z}{\partial t^2} = 0 \longrightarrow (15)$$

~~|||ly~~ This is required wave equation for electric field in plane polarized EM wave travelling in charge free, non conducting medium

|||ly taking $\frac{\partial}{\partial t}$ of eqⁿ 9 & $\frac{\partial}{\partial x}$ of eqⁿ 12 we have

$$\frac{\partial^2 E_z}{\partial x \partial t} = \frac{\partial^2 B_y}{\partial t^2}$$

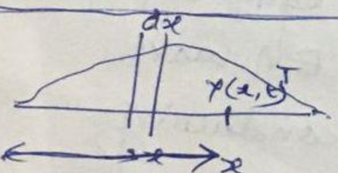
$$\frac{\partial^2 B_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial E_z}{\partial x \partial t}$$

From above eq^{ns} we can write

$$\frac{\partial^2 B_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_y}{\partial t^2}$$

$$\therefore \frac{\partial^2 B_y}{\partial x^2} - \mu_0 \epsilon_0 \frac{\partial^2 B_y}{\partial t^2} = 0 \longrightarrow (16)$$

This is wave eq^n for magnetic field in plane polarized EM wave travelling in charge free, non conducting medium.



$$y(x,t) = y(x - vt)$$

$$t = 0 \quad z = x - vt$$

$$(1) \quad y = F(z)$$

$$\frac{\partial y}{\partial x} = \frac{\partial F}{\partial z} \cdot \frac{\partial z}{\partial x} = \frac{\partial F}{\partial z} \qquad \Bigg| \qquad \frac{\partial y}{\partial t} = \frac{\partial F}{\partial z} \cdot \frac{\partial z}{\partial t} = \frac{\partial F}{\partial z}(-v)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 F}{\partial z^2} \qquad \Bigg| \qquad \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 F}{\partial z^2}(-v)(-v)$$

$$\frac{\partial^2 y}{\partial t^2} = v^2 \cdot \frac{\partial^2 y}{\partial x^2}$$

$$\boxed{\therefore \frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} = 0}$$

Transverse nature of E.M. wave

As the wave propagates along x-direction, then $E_x = 0$ & $B_x = 0$. In other words, electromagnetic wave have no longitudinal component. Further variation of $\bar{B}$ & $\bar{E}$ occurs only in direction $\perp^r$ to x-axis i.e. direction of propagation. From eq^n (9) to (12) we see that neither time derivative nor x-derivative of E_y, E_z (or B_y B_z) vanish. The transverse nature of EM waves can be proved by dividing eq^n (9) by eq^n (10)

$$\frac{\frac{\partial E_z}{\partial x}}{\frac{\partial E_y}{\partial x}} = \frac{\frac{\partial B_y}{\partial t}}{-\frac{\partial B_z}{\partial t}} \qquad \therefore \frac{\partial E_z}{\partial E_y} = -\frac{\partial B_y}{\partial B_z}$$

$$\frac{E_z}{E_y} = -\frac{B_y}{B_z} \qquad E_z B_z + E_y B_y = 0 \longrightarrow (13)$$

Now $\bar{E} \cdot \bar{B} = EB\cos\theta$ where θ is angle between electric field $\bar{E}$ & magnetic field $\bar{B}$.

Now, $\bar{E} = iE_x + jE_y + kE_z$

& $\bar{B} = iB_x + jB_y + kB_z$

Now let us assume solution of wave eq^n for electric & magnetic field be

$$E = \hat{j} \underbrace{E_0 \sin(kx - \omega t)}_{E_y} \quad \& \quad B = \hat{k} \underbrace{B_0 \sin(kx - \omega t)}_{B_z}$$

From eq^n

$$\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t} \rightarrow (9) \qquad \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \rightarrow (10)$$

$$\frac{\partial B_z}{\partial x} = \cancel{+}\mu_0 \epsilon_0 \frac{\partial E_y}{\partial t} \rightarrow (11) \qquad \frac{\partial B_y}{\partial x} = \mu_0 \epsilon_0 \frac{\partial E_z}{\partial t} \rightarrow (12)$$

Use eq^n (10)

$$\cancel{\frac{\partial}{\partial x}\left[E_0 \sin(kx - \omega t)\right] = \frac{\partial}{\partial t}\left[B_0 \sin(kx - \omega t)\right]}$$

$$\cancel{k E_0 \cos(kx - \omega t) = (-\omega) B_0 \cos(kx - \omega t)}$$

~~$E_0 =$~~

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}.$$

$$k E_0 \cos(\cancel{kx - \omega t}) = -B_0(-\omega) \cos(\cancel{kx - \omega t})$$

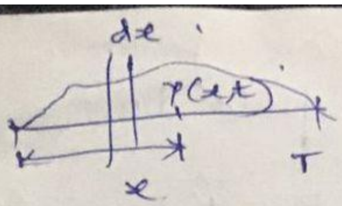
$$\boxed{\therefore E_0 = \frac{\omega}{k} B_0}$$

$$k B_0 \cos(\cancel{kx - \omega t}) = -\mu_0 \epsilon_0 \cancel{E_0} (-\omega) E_0 \cos(\cancel{kx - \omega t})$$

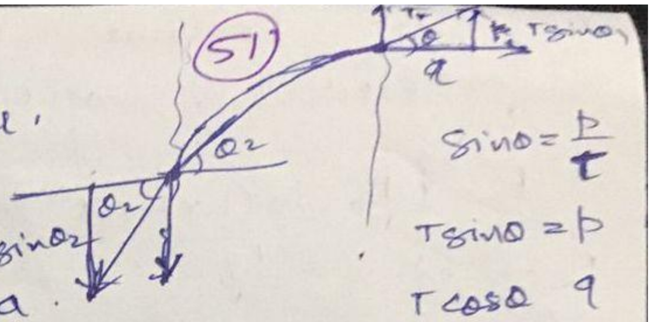
$$E_0 = \frac{k}{\mu_0 \epsilon_0 \omega} B_0 = \frac{c^2 k}{\omega} B_0 \quad \therefore \frac{\omega}{k} = \frac{c^2 k}{\omega}$$

$$\boxed{E_0 = \frac{c^2 k}{\omega} B_0} \qquad \omega^2 = c^2 k^2$$

$$\omega = \pm kc$$



μ.



$\sin\theta = \frac{p}{T}$

$T\sin\theta = p$

$T\cos\theta \; q$

$\tan\theta = \frac{p}{q}$

$\sin\theta = \theta - \frac{\theta^3}{3!} + \cdots$

$\cos\theta = 1 - \frac{\theta^2}{2} + \cdots$

$\tan\theta = \theta + \cdots$

$$T\sin\theta_1 - T\sin\theta_2 = ma.$$

$$T(\sin\theta_1 - \sin\theta_2) = \mu \cdot dx \cdot \frac{\partial^2 \varphi}{\partial t^2}$$

$$T(\tan\theta_1 - \tan\theta_2)$$

$$T\left(\left.\frac{\partial \varphi}{\partial x}\right|_{x+dx} - \left.\frac{\partial \varphi}{\partial x}\right|_{x}\right)$$

$$T\left(\frac{\partial}{\partial x}\left(\frac{\partial \varphi}{\partial x}\right) \cdot dx\right) = \mu \cdot dx \frac{\partial^2 \varphi}{\partial t^2}$$

$$T\frac{\partial^2 \varphi}{\partial x^2} \cdot \cancel{dx} = \mu \cancel{dx} \frac{\partial^2 \varphi}{\partial t^2}$$

$$\frac{\partial^2 \varphi}{\partial x^2} - \left(\frac{\mu}{T}\right)\frac{\partial^2 \varphi}{\partial t^2} = 0 \qquad \frac{1}{v^2} = \frac{\mu}{T}$$

$$\boxed{v = \sqrt{\frac{T}{\mu}}.}$$

$$v^2 = \frac{1}{\mu_0 \epsilon_0} = \frac{1}{4\pi\epsilon_0} \times \frac{4\pi}{\mu_0} = 9\times 10^9 \times 10^7 = 9\times 10^{16}$$

$\therefore v = 3\times 10^8$ m/s.